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TEMPERATURE FIELD IN A HALFSPACE WITH A PARALLELEPIPED-SHAPED HEAT-RELEASING INCLUSION

Yu. M. Kolyano, Yu. M. Krichevets,
E. G. Ivanik, and V. I. Gavrysh

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A study is made of the stationary temperature field in a half-space containing a foreign heat-releasing inclusion of parallelepiped shape of small dimensions.

In the operation of metalloceramic bodies of radio-electronic apparatus a need arises for studying temperature fields for bodies with foreign inclusions of small dimensions.

In this connection we consider an isotropic halfspace containing, at a distance ℓ from its boundary surface, a foreign inclusion of parallelepiped shape and volume $V_0 = 8hbd$ in whose vicinity uniformly distributed internal heat sources of strength q_0 are operative. We refer the body in question to a rectangular cartesian coordinate system. We place the coordinate origin at the center of the inclusion. On the boundary surface $z = -d-\ell$ a convective heat exchange is specified with external mean temperature t_c.

For the determination of the stationary temperature field we have the heat conduction equation [1]

$$\frac{\partial}{\partial x}\left[\lambda(x, y, z)\frac{\partial\Theta}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(x, y, z)\frac{\partial\Theta}{\partial y}\right]+ +\frac{\partial}{\partial z}\left[\lambda(x, y, z)\frac{\partial\Theta}{\partial z}\right]=-Q(x, y, z), \quad (1)$$

where

$$\lambda(x, y, z)=\lambda_1+(\lambda_0-\lambda_1)N(x, h)N(y, b)N(z, d);$$
$$Q(x, y, z)=q_0N(x, h)N(y, b)N(z, d); \quad (2)$$
$$\Theta=t-t_c;\ N(x, h)=S(x+h)-S(x-h).$$

The boundary conditions may be written in the form

$$\lambda_1\frac{\partial\Theta}{\partial z}=\alpha_z\Theta \text{ for } z=-d-l,\ \Theta=0 \text{ for } z\to\infty,$$
$$\Theta=0,\ \frac{\partial\Theta}{\partial x}=0 \text{ for } |x|\to\infty,\ \Theta=0,\ \frac{\partial\Theta}{\partial y}=0 \text{ for } |y|\to\infty. \quad (3)$$

We assume that the dimensions of the foreign inclusion are small in comparison with the distance from the coupling boundary to the boundary surface. We introduce the adduced thermal conductivity $\Lambda_0 = \lambda_0V_0$ of the inclusion, the adduced power $Q_0 = q_0V_0$ of the heat sources acting in it, and in Eqs. (2) we pass to the limit, letting $h \to 0$, $b \to 0$, $d \to 0$,

Institute of Applied Problems of Mechanics and Mathematics, Academy of Sciences of the UkrSSR, L'vov. Scientific and Engineering Institute of Materials, L'vov. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 57, No. 5, pp. 849-853, November, 1989. Original article submitted May 24, 1988.

maintaining Λ_0 and Q_0 constant, and using the well-known [2] limit $\lim_{h\to 0}\frac{N(x,h)}{2h}=\delta(x)$. As a result we have

$$\lambda(x,y,z)=\lambda_1+\Lambda_0\delta(x,y,z), \tag{4}$$

$$Q(x,y,z)=Q_0\delta(x,y,z). \tag{5}$$

Although the local non-homogeneity described by the relation (4) containing the Dirac delta-function is formally concentrated at the origin, it is, in fact, characterized by finite dimensions associated with the volume V_0. Thus the finite dimensions of the inclusion are effectively accounted for by relation (4) (see [3]).

Substituting expressions (4) and (5) into Eq. (1), we obtain the eqaution

$$\Delta\Theta+\frac{\Lambda_0}{\lambda_1}\left[\left.\frac{\partial\Theta(x,0,0)}{\partial x}\right|^{*}_{x=0}\delta'(x)\delta(y,z)+\left.\frac{\partial\Theta(0,y,0)}{\partial y}\right|^{*}_{y=0}\delta'(y)\delta(x,z)+\right.$$
$$\left.+\left.\frac{\partial\Theta(0,0,z)}{\partial z}\right|^{*}_{z=0}\delta'(z)\delta(x,y)\right]=-\frac{Q_0}{\lambda_1}\delta(x,y,z), \tag{6}$$

where

$$\left.\frac{\partial\Theta(x,0,0)}{\partial x}\right|^{*}_{x=0}=\frac{1}{2}\left[\left.\frac{\partial\Theta(x,0,0)}{\partial x}\right|_{x=+0}+\left.\frac{\partial\Theta(x,0,0)}{\partial x}\right|_{x=-0}\right].$$

Applying the Fourier integral transform with respect to coordinates x and y to equation (6) and conditions (2), we arrive at the following boundary value problem:

$$\frac{d^2\bar{\Theta}}{dz^2}-\gamma^2\bar{\Theta}=-P_1\delta(z)-P_2\delta'(z), \tag{7}$$

$$\lambda_1\frac{d\bar{\Theta}}{dz}=\alpha_z\bar{\Theta}\ \text{for}\ z=-d-l,\ \bar{\Theta}=0\ \text{for}\ z\to\infty, \tag{8}$$

where

$$\bar{\Theta}=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp i\alpha x dx\int_{-\infty}^{\infty}\Theta\exp i\beta y dy;\ \gamma^2=\alpha^2+\beta^2;$$

$$P_1=\frac{Q_0}{2\pi\lambda_1};\ P_2=\frac{\Lambda_0}{2\pi\lambda_1}\left.\frac{\partial\Theta(0,0,z)}{\partial z}\right|^{*}_{z=0}.$$

Here we have taken into account that $\left.\frac{\partial\Theta(x,0,0)}{\partial x}\right|^{*}_{x=0}=\left.\frac{\partial\Theta(0,y,0)}{\partial y}\right|^{*}_{y=0}=0.$

The solution of Eq. (7), subject to conditions (8), has the form

$$\bar{\Theta}=\frac{1}{2}\left\{\frac{P_1}{\gamma}[F_1(z)-F_2(z+d_1)]-P_2[F_1(z)\operatorname{sign}z+F_2(z+d_1)]\right\}, \tag{9}$$

where

$$F_1(z)=\exp(-\gamma|z|);\ F_2(z)=\frac{\alpha_z-\lambda_1\gamma}{\alpha_z+\lambda_1\gamma}\exp(-\gamma z);\ d_1=2(d+l).$$

Performing the inverse transform on Eq. (9) and using the reference data from [4, 5], we arrive at an expression for the dimensionless temperature

$$T=\frac{\lambda_1 l}{Q_0}\Theta=\frac{1}{4\pi}[\varphi^{-\frac{1}{2}}(X,Y,Z)+\varphi^{-\frac{1}{2}}(X,Y,Z+D)-$$
$$-\mathrm{Bi}\,\psi(X,Y,Z+D)]+\frac{1}{2}P_2[Z\varphi^{-\frac{3}{2}}(X,Y,Z)- \tag{10}$$
$$-(Z+D)\varphi^{-\frac{3}{2}}(X,Y,Z+D)+\mathrm{Bi}\,\psi_1(X,Y,Z+D)],$$

where

$$X=\frac{x}{l};\ Y=\frac{y}{l};\ Z=\frac{z}{l};\ D=\frac{d_1}{l};\ \varphi(X,Y,Z)=X^2+Y^2+Z^2;$$

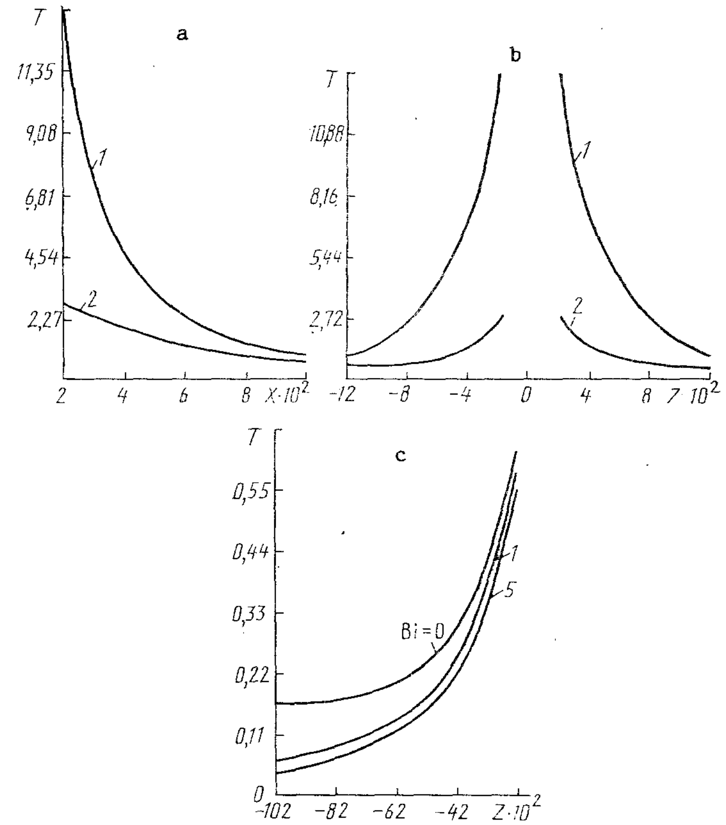

Fig. 1. Dependence of dimensionless temperature T on: (a) dimensionless coordinate X for Bi = 1, Z = 0.02; (b) dimensionless coordinate Z for Bi = 1, X = 0.02; (c) dimensionless coordinate Z for X = 0.02.

$$\psi(X,\ Y,\ Z) = 2\exp \mathrm{Bi}\, Z \int\limits_{Z}^{\infty} \exp(-\mathrm{Bi}\, Z)\, \varphi^{-\frac{1}{2}}(X,\ Y,\ Z)\, dZ;$$

$$\psi_1(X,\ Y,\ Z) = 2\exp \mathrm{Bi}\, Z \int\limits_{Z}^{\infty} Z\exp(-\mathrm{Bi}\, Z)\, \varphi^{-\frac{3}{2}}(X,\ Y,\ Z)\, dZ;$$

$$\left.\frac{\partial\Theta(0,\ 0,\ z)}{\partial z}\right|^{*}_{z=0} = \left\{\frac{1}{d^2} + \frac{1}{4(d+l)^2} + \frac{\mathrm{Bi}}{l}\left[\frac{\mathrm{Bi}}{l}\psi(0,\ 0,\ D) - \frac{1}{l+d}\right]\right\} \Bigg/ \left\{l\left[4\pi + \frac{\Lambda_0}{\lambda_1}\left(\frac{1}{4(l+d)^3} - \frac{\mathrm{Bi}}{2l(d+l)^2} + \frac{\mathrm{Bi}^2}{l^3}\right)\psi_1(0,\ 0,\ D)\right]\right\}.$$

For Bi = 0 expression (10) has the form

$$T = \frac{1}{4\pi}\left[\varphi^{-\frac{1}{2}}(X,\ Y,\ Z)\left(1 + \frac{1}{2}P_2 Z\varphi^{-1}(X,\ Y,\ Z)\right) + \varphi^{-\frac{1}{2}}(X,\ Y,\ Z+D)\left(1 - \frac{1}{2}P_2(Z+D)\varphi^{-1}(X,\ Y,\ Z+D)\right)\right]. \tag{11}$$

Here $\left.\frac{\partial\Theta(0,\ 0,\ z)}{\partial z}\right|^{*}_{z=0} = \left(\frac{1}{d^2} - \frac{1}{4(d+l)^2}\right) \Big/ \left[l\left(4\pi + \frac{\Lambda_0}{4\lambda_1(d+l)^3}\right)\right].$

Using formulas (10) and (11) with Y = 0, calculations were made and a study was conducted of the dimensionless temperature distribution T for the following initial data:

TABLE 1. Dependence of Dimensionless Temperature T on Dimensionless Coordinate Z for X = 0.02

$-Z\cdot10^2$	Bi			$-Z\cdot10^2$	Bi		
	0	1	5		0	1	5
22	0,6502	0,5924	0,5769	10	1,9731	1,9177	1,9038
18	0,8498	0,7928	0,7781	6	4,2007	4,1460	4,1324
14	1,2075	1,1514	1,1371	2	13,6778	13,6231	13,6094

basic material, ceramic BK94-1; inclusion material, molybdenum; $h/\ell = b/\ell = d/\ell = 0.02$. Numerical results illustrating the variation of the dimensionless temperature along coordinate X for Z = 0.02 and along coordinate Z for X = 0.02 are shown in Fig. 1; values of the dimensionless temperature for $-22 \le z\cdot10^2 \le -2$ and Bi = 0; 1; 5 are given in Table 1.

It is evident from Figs. 1a and 1b that the temperature increases monotonically with decrease in the values of X and $|Z|$ and attains its largest value in the region of operation of the heat sources (curve 1 in the case of a foreign inclusion in the halfspace; curve 2 in the case of a homogeneous halfspace). It is seen that presence of a foreign heat-releasing inclusion leads to a significant increase in the temperature. In the region $|Z| \le 0.12$ a symmetry may be observed in the temperature field with respect to the plane Z = 0.

Figure 1c illustrates the dependence of temperature T on coordinate Z for various values of the Biot number. It is evident that with an increase in heat emission the temperature diminishes.

NOTATION

$T(x, y, z)$, temperature field; $\lambda(x, y, z)$, thermal conductivity coefficient of nonhomogeneous body; λ_1, λ_0, thermal conductivity coefficients of basic material and of the inclusion; α_z, coefficient of heat transfer from the surface $z = -\ell - d$; $S(\zeta)$; symmetric unit function; $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$, Laplace operator; $\delta(\zeta)$, Dirac delta function; $Bi = (\alpha_2\ell)/\lambda_1$, Biot number.

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